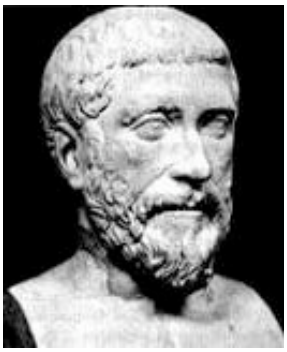


Pythagoras
[Samos, 582 - 500 BC]

Like Thales, Pythagoras is rather known for mathematics than for philosophy. Anyone who can recall math classes will remember the first lessons of plane geometry that usually start with the Pythagorean theorem about right-angled triangles: $a^2+b^2=c^2$. In spite of its name, the Pythagorean theorem was not discovered by Pythagoras. The earliest known formulation of the theorem was written down by the Indian mathematician Baudhāyana in 800BC. The principle was also known to the earlier Egyptian and the Babylonian master builders. However, Pythagoras may have proved the theorem and popularised it in the Greek world. With it, his name and his philosophy have survived the turbulences of history.

His immediate followers were strongly influenced by him, and even until today Pythagoras shines through the mist of ages as one of the brightest figures of early Greek antiquity. The Pythagorean theorem is often cited as the beginning of mathematics in Western culture, and ever since mathematics - the art of demonstrative and deductive reasoning- has had a profound influence on Western philosophy, which can be observed down to Russell and Wittgenstein.

Pythagoras' influence found an expression in visual art and music as well, particularly in the renaissance and baroque epoch. The far-reaching imprint of his ideas is yet more impressive if we consider that he did not leave any original writings. Instead, all what is known about Pythagoras was handed down by generations of philosophers and historiographers, some of whom, like Heraclitus, opposed his views. In this light it is remarkable that Pythagoras' teachings have survived relatively undistorted until the present day.

Pythagoras was a native of the island of Samos. During his early life, Samos was governed by the powerful, unscrupulous tyrant Polycrates. Pythagoras did not sympathise with his government and thus emigrated to Croton in Southern Italy. Like the ancient Greek cities in Ionia, Croton was a flourishing commercial city that lived from importing and exporting goods. Obviously it was in Croton where Pythagoras developed most of

his important ideas and theories.

Pythagoras founded a society of disciples which has been very influential for some time. Men and women in the society were treated equally -an unusual thing at the time- and all property was held in common. Members of the society practised the master's teachings, a religion the tenets of which included the transmigration of souls and the sinfulness of eating beans. Pythagoras' followers had to obey strict religious orders where it was forbidden to eat beans, to touch white cocks, or to look into a mirror beside a light.

If all of this seems a bit odd, it might lead us to suspect that Pythagoras' personality reflects the inseparable blend of genius and madness that we associate with many other great men. It is said that once Pythagoras was walking up a lane in Croton when he came by a dog being ill-treated. Seeing this he raised his voice: "Stop, don't hit it! It is a soul of a friend. I knew it when I heard its voice." Spirits, ghosts, souls, and transmigration were obviously things he believed in deeply.

There was an opposition -if not rivalry- in ancient Greece between the gods of the Olympus and the lesser gods of more primitive religions. Pythagoras, like no other, embodied the contradistinctions of the mystical and rational world, which is woven into his personality and philosophy. In his mind, numbers, spirits, souls, gods and the mystic connections between them formed one big picture. The following text tells the legend of his own existences:

"He was once born as Aethalides and was considered to be the son of Hermes. Hermes invited him to choose whatever he wanted, except immortality; so he asked that, alive and dead, he should remember what happened to him. Thus, in life he remembered everything, and when he died he retained the same memories. [...] He remembered everything - how he first had been Aethalides, then Euphorbus, then Hermotimus, then Pyrrhus, the Delian fisherman. When Pyrrhus died, he became Pythagoras." (Diogenes Laertius, Live of Philosophers, VIII 4-5)

"Pythagoras believed in metempsychosis and thought that eating meat was an abominable thing, saying that the souls of all animals enter different animals after death. He himself used to say that he remembered being, in Trojan times, Euphorbus, Panthus' son who was killed by Menelaus. They say that once when he was staying at Argos he saw a shield from the spoils of Troy nailed up, and burst into tears. When the Argives asked him the reason for his emotion, he said that he himself had borne that shield at Troy when he was Euphorbus.

They did not believe him and judged him to be mad, but he

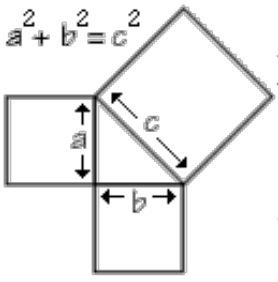
said he would provide a true sign that it was indeed the case: on the inside of the shield there had been inscribed in archaic lettering EUPHORBUS. Because of the extraordinary nature of his claim they all urged that the shield be taken down - and it turned out that on the inside the inscription was found." (Diogenes Laertius)

After Pythagoras introduced the idea of eternal recurrence into Greek thought, which was apparently motivated by his studies of earlier Egyptian scriptures, the idea soon became popular in Greece. It was Pythagoras' ambition to reveal in his philosophy the validity and structure of a higher order, the basis of the divine order, for which souls return in a constant cycle.

This is how Pythagoras came to mathematics. It could be said that Pythagoras saw the study of mathematics as a purifier of the soul, just like he considered music as purifying. Pythagoras and his disciples connected music with mathematics and found that intervals between notes can be expressed in numerical terms. They discovered that the length of strings of a musical instrument correspond to these intervals and that they can be expressed in numbers. The ratio of the length of two strings with which two tones of an octave step are produced is 2:1.

Music was not the only field that Pythagoras considered worthy of study, in fact he saw numbers in everything. He was convinced that the divine principles of the universe, though imperceptible to the senses, can be expressed in terms of relationships of numbers. He therefore reasoned that the secrets of the cosmos are revealed by pure thought, through deduction and analytic reflection on the perceptible world.

This eventually led to the famous saying that "all things are numbers." Pythagoras himself spoke of square numbers and cubic numbers, and we still use these terms, but he also spoke of oblong, triangular, and spherical numbers. He associated numbers with form, relating arithmetic to geometry. His greatest contribution, the proposition about right-angled triangles, sprang from this line of thought:



"The Egyptians had known that a triangle whose sides are 3, 4, 5 has a right angle, but apparently the Greeks were the first to observe that $3^2+4^2=5^2$, and, acting on this suggestion, to discover a proof of the general proposition. Unfortunately for Pythagoras this theorem led at once to the discovery of incommensurables, which appeared to disprove his whole philosophy. In a right-angled isosceles triangle, the square on the hypotenuse is double of the square on either side.

Let us suppose each side is an inch long; then how long is the hypotenuse? Let us suppose its length is m/n inches. Then $m^2/n^2=2$. If m and n have a common factor, divide it out, then either m or n must be odd. Now $m^2=2n^2$, therefore m^2 is even, therefore m is even, therefore n is odd. Suppose m=2p. Then $4p^2=2n^2$, therefore $n^2=2p^2$ and therefore n is even, contra hyp. Therefore no fraction m/n will measure the hypotenuse. The above proof is substantially that in Euclid, Book X." (Bertrand Russell, History of Western Philosophy)

This shows how Pythagoras' formulation immediately led to a new mathematical problem, namely that of incommensurables. At his time the concept of irrational numbers was not known and it is uncertain how Pythagoras dealt with the problem. We may surmise that he was not too concerned about it. His religion, in absence of theological explanations, had found a way to blend the "mystery of the divine" with common-sense rational thought.

From Pythagoras we observe that an answer to a problem in science may give raise to new questions. For each door we open, we find another closed door behind it. Eventually these doors will be also be opened and reveal answers in a new dimension of thought. A sprawling tree of progressively complex knowledge evolves in such manner. This Hegelian recursion, which is in fact a characteristic of scientific thought, may or may not have been obvious to Pythagoras. In either way he stands at the beginning of it.